

Synthetic Polynomial Division for Non-Monic Divisors

1. Abstract

Polynomial synthetic division is a faster way to complete the division of polynomials using a factor or another non-linear expression (e.g $x-c$). This avoids the complexity of other methods such as long division or inspection, and it serves as a shortcut. The primary weakness of synthetic division is that it cannot use non-monic linear expressions (e.g $2x-c$) as a divisor. This paper covers an approach in order to overcome this obstacle with synthetic division.

2. Introduction

Most information used to achieve these adjustments were from the Cambridge VCE Units 1 and 2 Mathematical Methods 2023 textbook, within the polynomials chapter where long division was covered. Due to the incapability of synthetic division to use non-monic linear divisors under normal circumstances, this led to further testing using various synthetic division problems from the textbook, and under various manipulations of the methods until a method was

3. Method 1: Conversion into monic divisor

Given a divisor in the form $ax-c$, we can factor out a and rewrite the divisor as:

$$ax-c = a\left(x-\frac{c}{a}\right)$$

This allows us to divide the entire polynomial by a first and then perform synthetic division with $x=\frac{c}{a}$

Example:

Divide $2x^3+3x^2-32x+15$ by $2x-1$

Step 1: Factor the divisor

$$2x-1 = 2\left(x-\frac{1}{2}\right)$$

Step 2: Divide the polynomial by 2 first:

$$\frac{2x^3+3x^2-32x+15}{2} = x^3 + \frac{3}{2}x^2 - 16x + \frac{15}{2}$$

Step 3: Apply synthetic division using $x=\frac{1}{2}$

$\frac{1}{2}$	1	$\frac{3}{2}$	-16	$\frac{15}{2}$
		$\frac{1}{2}$	1	$-\frac{15}{2}$

	1	2	-15	0
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This gives the quotient:

$$x^2 + 2x - 15 \text{ with no remainder}$$

4. Conclusion

For non-monic divisors in a synthetic division problem, the polynomial can be divided by the coefficient of x and then it can be factored out of the divisor itself, then synthetic division can be done in order facilitate the conditions for synthetic division to work, as under normal circumstances it must have a monic linear divisor.